

Bush encroachment control and risk management in semi-arid rangelands

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Abstract: We study the role of bush encroachment control for a farmer's income and income risk in a stochastic ecological-economic model of grazing management in semi-arid rangelands. In particular, we study debushing as an instrument of risk management that complements the choice of an adaptive grazing management strategy for that sake. We show that debushing, while being a good practice for increasing the mean pasture productivity and thus expected income, also increases the farmer's income risk. The optimal extent of debushing for a risk-averse farmer is thus determined from balancing the positive and negative consequences of debushing on intertemporal and stochastic farm income.

JEL-Classification: Q57, Q15

Keywords: bush encroachment, expected utility, farm income, intertemporal optimization, risk aversion, risk management, semi-arid rangeland

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1 Introduction

Bush encroachment is considered to be one of the most extensive forms of degradation in rangelands in arid and semi-arid regions of the Earth (Sweet 1998, de Klerk 2004, Joubert et al. 2009, Schröter et al. 2010). With arid and semi-arid areas covering about one quarter of the land surface of the Earth, between 50 and 80% of these areas being used as rangelands, and more than one billion people earning their livelihood directly from livestock farming in these areas (Millennium Ecosystem Assessment 2005), bush encroachment is a major worldwide problem. In Namibia, for example, where the economic well-being of more than two thirds of the population depends directly or indirectly on agriculture and 65% of the national agricultural output is produced on commercial rangeland (Mendelssohn et al. 2003), bush encroachment severely restricts profitability of cattle farming (Espach 2006); the same goes for South Africa (Stuart-Hill 1987, Börner et al. 2007) or Uganda (Mugasi et al. 2000).

From an ecological-economic point of view, rangelands in (semi-)arid regions are savannahs that are characterized by dynamic interaction and coexistence of woody and herbaceous vegetation, i.e. bushes and grass, under the influence of stochastic precipitation and bushfire, and that are managed for the purpose of livestock grazing (Knoop and Walker 1985, Perrings and Walker 1997, Wiegand and Jeltsch 2000, Beukes et al. 2002, Sullivan and Rohde 2002, Janssen et al. 2004, Riginos 2009). The crucial ecosystem service that limits livestock production and shapes farming strategies, is production of green grass biomass, which serves as a forage for livestock and thus generates farm income.

Low and highly variable precipitation, which is typical in (semi-)arid regions, causes a considerable income risk for farmers. The challenge of rangeland management is to optimally adapt to this highly variable and highly uncertain rainfall, taking into account ecosystem dynamics. Various grazing management strategies have been developed for that sake (Westoby et al. 1989, Behnke et al. 1993, Scoones 1994, Heady 1999, Rothauge 2007, Hein and Weikard 2008, Weikard and Hein in press). One such strategy, which is applied in many good-practice farms in Southern Africa, is to leave a fixed part of the

pasture ungrazed in years with abundant rainfall (“resting in rainy years”), i.e. stocking is less than the grazing capacity of the pasture in such a year, while the pasture is used fully in years with low precipitation. Such conservative grazing management has been shown to be an efficient strategy for income risk reduction (Müller et al. 2007, Quaas et al. 2007, Müller et al. 2009, Quaas and Baumgärtner 2010).

Ill-adapted grazing management strategies, including over-stocking and suppression of bushfires, are the major anthropogenic causes of bush encroachment, i.e. the persisting occurrence of an ecosystem state dominated by woody vegetation (Roques et al. 2001, de Klerk 2004, Joubert et al. 2008).¹ Bush encroachment leads to a reduction in the production of green grass biomass and, thus, to a reduction of grazing capacity of the rangeland (Sweet 1998, de Klerk 2004, Espach 2006). As a consequence, farm income is diminished.

Bush encroachment control aims at increasing the long-term carrying capacity of the pasture through physical, chemical or biological eradication of excessive woody biomass (“debushing”).² Generally, as in a savannah system there are not only negative but also some positive bush-grass interactions (Knoop and Walker 1985, Smit 2005), there is an optimal density of bushes that makes for the maximum carrying capacity of the rangeland (de Klerk 2004).³ Hence, debushing is not aimed at complete eradication of woody biomass, but at reduction of bush density down to the optimal level. As a result of debushing down to this optimal level, grass biomass production increases significantly in the first year (Wölbling 2008). Yet, bush encroachment sets in again after three to

¹Extreme droughts and climatic change are among the natural causes of bush encroachment.

²Indirect management practices, such as decreasing the stocking rate of livestock in order to recover the grass cover require a much longer time than direct physical or chemical measures (Valone et al. 2002) and are therefore hardly used.

³For instance, for Namibia this optimal density of bushes can be estimated from the following rule of thumb (de Klerk 2004: 60): two times the long-term average rainfall (measured in millimeters per year) in an area yields the optimal number of tree equivalents per hectare in that area, where a *tree equivalent* is defined as a tree (shrub) measuring 1.5 m in height (so that e.g. a 3-m shrub would represent 2 tree equivalents); for example, with a long-term average rainfall of 200 mm/a, some 400 tree equivalents per hectare would be optimal.

five years and grass biomass production drops. If a second round of complete debushing is then applied after some ten years, the effect of debushing persists for some twenty years (Wölbling 2008, Krüger and Lubbe 2010).

While the expectation is that, at bottom line, debushing increases a farmer's income, the exact effect of debushing on the intertemporal stream of farm income and, in particular, on the variability of income, has not been studied so far. In this paper, we study the role of debushing for a farmer's income and income risk in a stochastic ecological-economic model of grazing management in semi-arid rangelands. In particular, we study debushing as an instrument of risk management that complements the choice of an adaptive grazing management strategy for that sake.

We show that debushing, while being a good practice for increasing the mean pasture productivity and thus expected income, also increases the farmer's income risk. The optimal extent of debushing for a risk-averse farmer is thus determined from balancing the positive and negative consequences of debushing on intertemporal and stochastic farm income.

The paper is organized as follows. In Section 2, we present the stochastic and dynamic ecological-economic model, incorporating grazing management and debushing strategies. Section 3 describes the concepts and tools applied in the model evaluation. Section 4 presents the results of the study. Section 5 provides a discussion of these results and draws conclusions.

2 Model

Our analysis is based on an integrated dynamic and stochastic ecological-economic model which is generic in that it captures essential and general principles of livestock grazing management in (semi-)arid regions. The basic model was developed in previous analyses of good-practice examples, in particular Karakul sheep farming in Namibia (Müller et al. 2007, Quaas et al. 2007, Baumgärtner and Quaas 2009, Müller et al. 2009). In this model, we include here a stylized description of debushing. The basic structure of the model is presented in Figure 1.

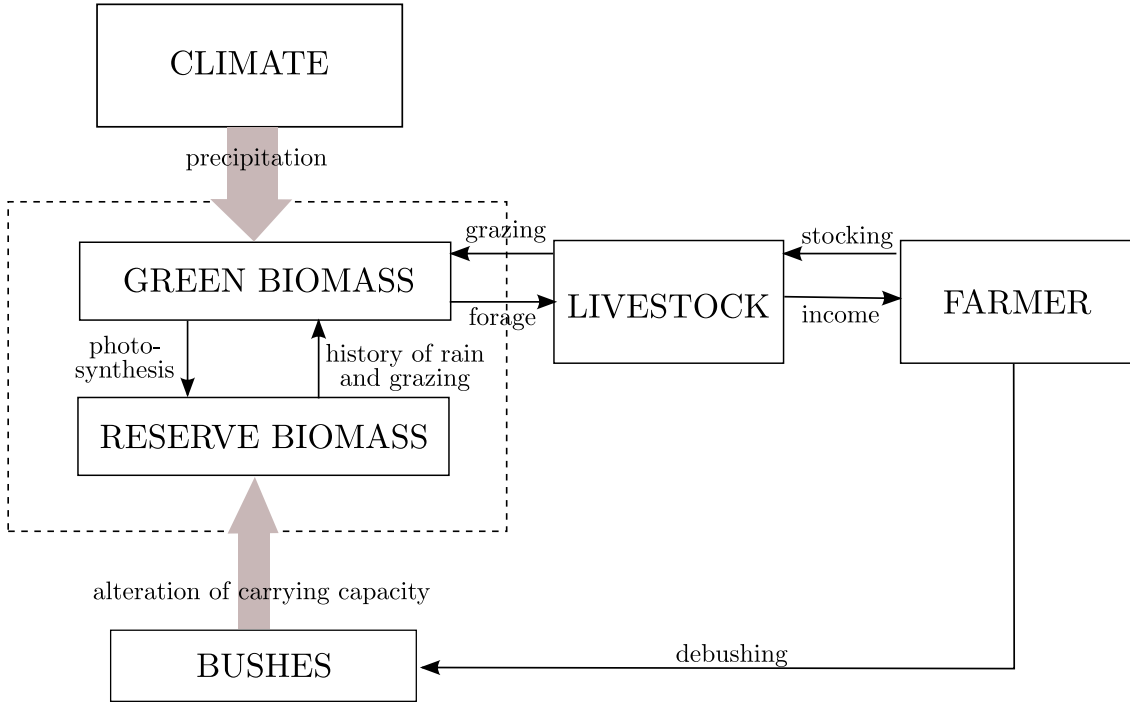


Figure 1: Basic structure of the model.

Precipitation

The essential exogenous driver of vegetation and livestock dynamics in semi-arid regions, which introduces uncertainty into the system, is precipitation. Precipitation is modeled as an independently and identically log-normally distributed random variable r (Sandford 1982) with mean $\mathbb{E}[r]$ and standard deviation $Sd[r]$. The probability density function is

$$f(r) = \frac{1}{r\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln r - \mu)^2}{2\sigma^2}\right), \quad (1)$$

$$\text{with } \mu = \ln \mathbb{E}[r] - \frac{1}{2} \ln(1 + Sd[r]^2/\mathbb{E}[r]) , \quad (2)$$

$$\sigma^2 = \ln(1 + Sd[r]^2/\mathbb{E}[r]^2) . \quad (3)$$

The distribution of rainfall events is right-skewed: events with low rainfall are frequent, but eventually high-rainfall-events occur. Precipitation is measured in units of effective rain events per year, that is the number of rain events that are effective in triggering

plant growth.⁴ For convenience, a continuous scale is assumed (Müller et al. 2009).

Grazing management

Grazing management is assumed to follow a “resting in rainy years”-strategy, where the farmer fully stocks in normal or dry years and stocks below the maximum (that is, gives the pasture a “rest”) in years with high rainfall. Its key feature is that in dry years, a farmer uses the whole pasture, while in years with sufficiently high rainfall levels, a pre-specified fraction of it is rested. This grazing management strategy can generically be represented as (γ, \underline{r}) , where $\gamma \in [0, 1]$ denotes the fraction of pasture rested if rainfall exceeds the threshold value $\underline{r} \in [0, \infty)$ (Quaas et al. 2007, Müller et al. 2009, Baumgärtner and Quaas 2009, Quaas and Baumgärtner 2010). These two control variables determine the fraction g_t of pasture that is used for grazing in any given year t depending on actual rainfall r_t in that year:

$$g_t = \begin{cases} 1 & \text{if } r_t \leq \underline{r} & \text{(no resting in normal or dry years)} \\ 1 - \gamma & \text{if } r_t > \underline{r} & \text{(resting in rainy years)} \end{cases} . \quad (4)$$

The higher the fraction of resting γ and the lower the rain threshold \underline{r} , the more resting is applied, that is, the more *conservative* is the grazing management strategy (Müller et al. 2007, Quaas et al. 2007).

Debushing

Woody plants are a natural part of dynamic savannah systems, with positive and negative interactions between woody and herbaceous vegetation, i.e. bushes and grass. It has been shown that below some optimal density (which depends on long-term average rainfall),⁵ bushes may have a positive impact on grass growth, while an increase of bush

⁴For example, in the (semi-)arid rangeland system of Namibia with mean annual precipitation of 180 mm/a, rain events of more than 15 mm/day are effective in this sense.

⁵For example, in the (semi-)arid rangeland system of Namibia with mean annual precipitation of 180 mm/a, the optimal bush density is 360 tree equivalents per hectare according to the rule of thumb of de Klerk (2004: 60).

density above this level leads to significant suppression of green grass biomass production and, thus, the rangeland's grazing capacity for livestock (de Klerk 2004: 58–62, 111–126).

In line with these stylized facts, we measure bush encroachment as the excess amount B of bushes over the optimal amount of bushes that yields the highest carrying capacity K (namely for $B = 0$). For $B > 0$ there is a negative relationship between bush encroachment B and carrying capacity of the pasture (Figure 2). If, for example, the

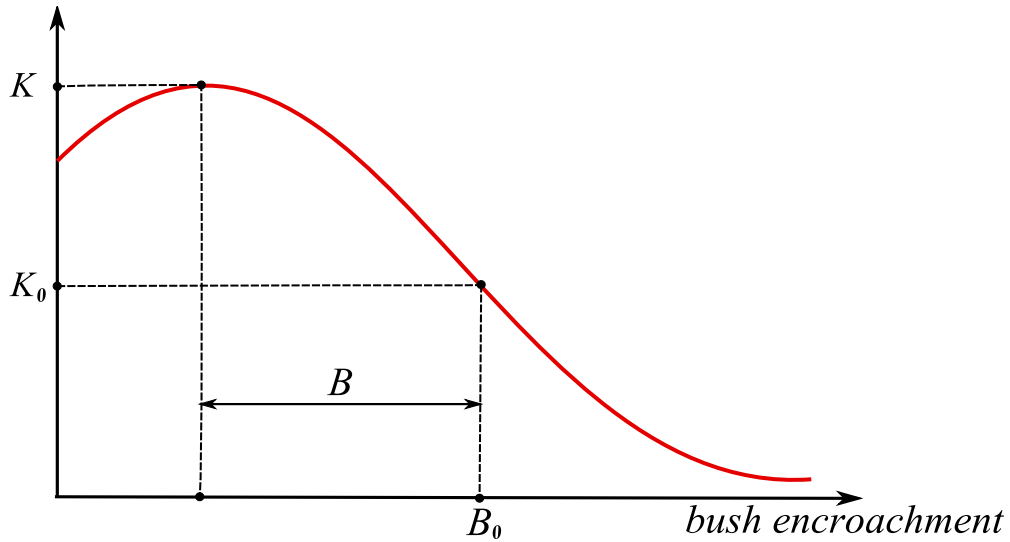


Figure 2: Dependence of carrying capacity on the amount of bushes (based on Stuart-Hill (1987) and de Klerk (2004: 58–62, 111–126)). B denotes the excess amount of bushes over the optimal amount of bushes, K the maximal achievable carrying capacity (for $B = 0$), and B_0 and K_0 the initial amount of bushes and the initial carrying capacity, respectively.

initial level of bush encroachment is B_0 , which corresponds to a carrying capacity of $K_0 < K$, debushing, i.e. a decrease of B , increases the resulting carrying capacity.

We assume that the farmer can directly choose, as another management variable besides the grazing management strategy, the fraction $\kappa \in [0, 1]$ of bush removal, i.e. the fraction of the excess amount B of bushes over the optimal amount of bushes which is

actually removed. We assume a linear relationship between the amount of bush removal and resulting increase of carrying capacity of the pasture, and normalize units appropriately, so that $\kappa \cdot B$ is the increase in carrying capacity of the pasture. Furthermore, we assume that all debushing takes place in the first year and that the increased carrying capacity due to debushing persists at the initial level throughout the whole time horizon.

While debushing a fraction of κ of the excess amount of bushes over the optimal amount of bushes generates the benefit of increased carrying capacity of the pasture, it also implies costs. We assume that the farmer incurs annualized costs of $\kappa \cdot C$, where $C > 0$ are the constant marginal costs of debushing. Part of the annualized costs of debushing can be regarded as the down-payment of a *sui generis* loan, that a farmer takes up initially in order to pay for the initial one-time increase in carrying capacity.⁶ Another part of the annualized total costs of debushing consists of the average annual costs that a farmer faces for keeping the pasture's carrying capacity on the debushed level.

Ecosystem dynamics

Grass vegetation dynamics are modeled by two variables that describe the two components of a single (representative) species of perennial grass: green biomass G_t which describes the photosynthetic organs of the plants that serve as forage for livestock; and reserve biomass R_t which describes non-photosynthetic organs that do not serve as forage (Baumgärtner and Quaas 2009, Müller et al. 2009).

Figure 1 presents the interaction of all five principal components of pasture vegetation dynamics, namely of green and reserve biomass, climate, livestock and bushes. Livestock is determined by grazing on green biomass, thus influencing the vegetation dynamics. The quantity of green biomass in any given year is determined by the actual amount of precipitation in that year and by the reserve biomass that has accumulated under the rain history and grazing history in previous years. The dynamics of green and reserve biomass are also influenced by competition with woody vegetation, which may suppress

⁶In fact, many debushing measures in Namibia are financed by loans.

the growth of grass vegetation, thus decreasing the carrying capacity of the pasture for livestock.

The amount G_t of green biomass (flow variable) available in year t after the end of the growing season is given by

$$G_t = w_G \cdot r_t \cdot R_t , \quad (5)$$

where the parameter w_G is a conversion parameter, indicating the extent to which green biomass G_t responds to reserve biomass R_t and current plant-available precipitation r_t . The dynamics of the reserve biomass (stock variable) are described by the following stochastic difference equation:

$$\begin{aligned} R_{t+1} = R_t - d \cdot R_t \cdot \left(1 + \frac{R_t}{K - (1 - \kappa) \cdot B} \right) \\ + w_R \cdot w_G \cdot (1 - c \cdot g_t) \cdot r_t \cdot R_t \cdot \left(1 - \frac{R_t}{K - (1 - \kappa) \cdot B} \right) , \end{aligned} \quad (6)$$

where d is the constant intrinsic death rate, and w_R is the constant intrinsic growth rate, of reserve biomass. A density dependence of reserve biomass growth is captured by the factors containing the maximal achievable carrying capacity K : the higher the accumulated reserve biomass, the slower it grows. The parameter $c \in [0, 1]$ describes the factor by which reserve biomass growth is reduced due to grazing pressure.

Herd size and farm income

The farmer's annual income is given by the revenues from selling livestock products such as meat, milk, fur and wool. This income is assumed to arise in proportion to the number S_t of livestock on the farm in that year, with the current price for livestock products as the factor of proportionality. With G_t (Equation 5) as the amount of green biomass available in year t , and taking into account the grazing management strategy (γ, \underline{r}) which in year t leads to a fraction g_t (Equation 4) of available green biomass used for grazing, the total number of livestock (herd size) S_t in year t is given by:

$$S_t = g_t \cdot G_t . \quad (7)$$

Assuming that the price for livestock products is constant, and normalizing it appropriately, gross income in any year t simply equals the number S_t of livestock in that year. Net income y_t contains in addition the annual costs of debushing, with marginal costs C . Thus, net income y_t in year t is

$$y_t = S_t - \kappa \cdot C = g_t \cdot G_t - \kappa \cdot C . \quad (8)$$

Since the herd size S_t is a random variable, annual income y_t is also a random variable.

Farmer's preferences and behavior

The farmer's preferences over the uncertain stream of present and future income $\{y_t\}_{t=1}^T$ are described by the expected-intertemporal-utility function

$$U = \mathbb{E} \left[\sum_{t=1}^T \frac{u(y_t)}{(1 + \delta)^{t-1}} \right] , \quad (9)$$

where $u(y_t)$ is the instantaneous utility obtained in year t from actual current income y_t , and $\delta > 0$ is the farmer's utility discount rate. The higher δ , the more impatient is the farmer. The expectation operator $\mathbb{E}[\cdot]$ takes the mean over the probability distribution of all rainfall profiles over the period $t = 1, \dots, T$. For $u(\cdot)$, we assume a constant-relative-risk-aversion-utility function:

$$u(y_t) = \frac{y_t^{1-\theta} - 1}{1 - \theta} , \quad (10)$$

where $\theta > 0$ is the constant degree of relative risk aversion.⁷ The higher θ , the more risk averse is the farmer.

The farmer is assumed to maximize expected intertemporal utility U (Equations 9 and 10) over the three control variables $\gamma \in [0, 1]$, $\underline{r} \in [0, \infty)$ and $\kappa \in [0, 1]$, and subject to the dynamics of the coupled ecological-economic system:

$$\max_{\gamma, \underline{r}, \kappa} U \quad \text{subject to} \quad (4), (5), (6), (8) . \quad (11)$$

Parameter values

Table 1 shows the parameter values used in the numerical analysis. The values of the

⁷As function (10) is not defined for $\theta = 1$, we define $u(\cdot)$ by (10) for $\theta \neq 1$, and by the continuous extension of (10) for $\theta = 1$, which is $u(y_t) = \log y_t$.

Table 1: Parameter set and parameter values

	Parameters		Values
Ecological conditions	Growth rate of green biomass	w_G	1.2
	Growth rate of reserve biomass	w_R	0.2
	Mortality rate of reserve biomass	d	0.15
	Impact of grazing	c	0.5
	Initial reserve biomass	R_0	1.0
	Maximal attainable carrying capacity	K	48.0
	Excess amount of bushes	B	40.0
Climatic conditions	Mean annual rainfall	$\mathbb{E}[r]$	1.2
	Standard deviation of annual rainfall	$Sd[r]$	0.7
Economic parameters	Risk aversion	θ	2.0
	Discount rate	δ	0.1
	Time horizon	T	15
	Marginal annualized costs of debushing	C	0.04

ecological parameters w_G , w_R , d , c , R_0 , K and B follow Müller et al. (2007, 2009). The values of K and B have been modified from the values used by Müller et al. (2007, 2009) according to the assumption that, starting from the current state of the rangeland, complete debushing will increase the carrying-capacity for livestock by a factor of five (Wölbling 2008). The values of the climatic parameters $\mathbb{E}[r]$ and $Sd[r]$ are taken from Sandford (1982). The values of risk aversion θ , time horizon T and discount rate δ are conforming well to the model peculiarities, and are in the range of values found in an empirical survey of commercial cattle farmers in Namibia (Olbrich et al. 2009). The value of the marginal annualized costs C of debushing has been derived from information on the one-time costs of physical debushing being in the range of 200–800 Namibian dollars per hectare (Wölbling 2008, Horsthemke 2010, Krüger and Lubbe 2010), by annualizing over the time horizon T and transforming unities appropriately.

3 Concepts and tools of model evaluation

Solving the stochastic maximization problem (11) yields the optimal grazing management strategy $(\gamma^*, \underline{r}^*)$ and the optimal fraction of debushing (κ^*) . If the farmer applies these under stochastic precipitation, an ex-ante uncertain income stream $y = \{y_t\}_{t=1}^T$ will result. In the following, we describe what concepts we use to obtain and evaluate this uncertain income stream.

Expected present value of uncertain income stream

The expected present value Y_E of the uncertain income stream y is defined as

$$Y_E = \mathbb{E} \left[\sum_{t=1}^T \frac{y_t}{(1 + \delta)^{t-1}} \right]. \quad (12)$$

Here, we use the utility discount rate δ as an income discount rate.

Coefficient of variation of reserve biomass and income at the end of the time-horizon

As a measure of volatility, we calculate the coefficient of reserve-biomass variation CV_R and of annual-income variation CV_y at the end of the time-horizon T as the ratio of standard deviation to mean (i.e. expected) reserve biomass R_t (6) in year $t = T$ and mean (i.e. expected) income y_t (8) in year $t = T$, respectively,

$$CV_R = \frac{Sd[R_T]}{\mathbb{E}[R_T]} \quad \text{and} \quad (13)$$

$$CV_y = \frac{Sd[y_T]}{\mathbb{E}[y_T]}, \quad (14)$$

where R_t and y_t are obtained as the optimal solution to maximization problem (11) for a given time horizon T . When doing the maximization (11) only over the grazing management strategy (γ, \underline{r}) , but not over the extent of debushing κ , one obtains the coefficients of variation CV_R and CV_y as functions of κ . They are a measure of relative dispersion in probability distributions. Therefore, they can be regarded as a measure of relative objective risk of a certain extent of debushing κ . As the coefficient of varia-

tion does not take into account the farmer's subjective risk aversion, they fall short of adequately measuring the subjectively valued risk, which is relevant for decision-making

Intertemporal certainty equivalent of an uncertain income stream

In an intertemporal and stochastic setting, one can define the intertemporal certainty equivalent Y_{CE} of the uncertain income stream y as the present value of a certain and constant payment stream over the entire time horizon $t = 1, \dots, T$ which generates the same expected intertemporal utility as the uncertain income stream. With (9) and (10), the expected intertemporal utility U from an income stream y is given by

$$U = \mathbb{E} \left[\sum_{t=1}^T \frac{y_t^{1-\theta} - 1}{(1-\theta)(1+\delta)^{t-1}} \right]. \quad (15)$$

The certain and constant annual payment y_{CE} which generates the same intertemporal utility U (Equation 15) as the uncertain income stream is, thus, determined by

$$U = \sum_{t=1}^T \frac{y_{CE}^{1-\theta} - 1}{(1-\theta)(1+\delta)^{t-1}}, \quad (16)$$

which can be solved for⁸

$$y_{CE} = \left[1 + (1-\theta)U / \sum_{t=1}^T \frac{1}{(1+\delta)^{t-1}} \right]^{\frac{1}{1-\theta}}. \quad (17)$$

This is the amount of money that, when paid for sure in each year over the entire time horizon $t = 1, \dots, T$, generates the same intertemporal utility as the expected intertemporal utility from the uncertain income stream y . The intertemporal certainty equivalent Y_{CE} is then the present value of the certain and constant income stream $\{y_{CE}\}_{t=1}^T$:

$$Y_{CE} = \sum_{t=1}^T \frac{y_{CE}}{(1+\delta)^{t-1}} = y_{CE} \sum_{t=1}^T \frac{1}{(1+\delta)^{t-1}}, \quad (18)$$

where y_{CE} is defined by Equations (17) and (15).

⁸For $\theta = 1$, one has $y_{CE} = \exp \left[U / \sum_{t=1}^T \frac{1}{(1+\delta)^{t-1}} \right]$.

Intertemporal risk premium of an uncertain income stream

The intertemporal risk premium π of the uncertain income stream y can be defined as the difference between the expected net present value, Y_E , and the intertemporal certainty equivalent, Y_{CE} , of the uncertain income stream y :⁹

$$\pi = Y_E - Y_{CE} , \quad (19)$$

where Y_E and Y_{CE} are defined by Equations (12) and (18), respectively. The risk premium π values, in monetary terms, the risk associated with the uncertain income stream y compared to the certain and constant payment stream y_{CE} that generates the same expected intertemporal utility to a risk-averse farmer. With this understanding of the risk premium, one may say that debushing provides natural insurance if the risk premium π decreases with the fraction of debushing κ .

Total net value of debushing

While debushing may have an impact on the income *risk* of a risk-averse farmer, which is measured by the change in the risk premium π (Equation 19), it may also affect the *expected* income, Y_E (Equation 12). In order to measure the total net value of debushing to a risk-averse farmer, taking into account both effects, we therefore calculate the farmer's willingness to pay for debushing when debushing comes at annual marginal costs of C (hence: "net" value). A positive total net value indicates that the farmer would prefer debushing (at annual marginal costs C) over not debushing.

In order to calculate the farmer's willingness to pay for debushing, we compare the uncertain income stream $y = \{y_t\}_{t=1}^T$ that results from applying the optimal grazing management strategy $(\gamma^*, \underline{t}^*)$ and the optimal fraction of debushing (κ^*) with the uncertain income stream $y^0 = \{y_t^0\}_{t=1}^T$ that would result from applying the grazing management strategy $(\gamma^0, \underline{t}^0)$ that solves the stochastic maximization problem (11) if the

⁹Lau (2008) suggests a more demanding concept of "risk premium" in an intertemporal context, based on an attempt to explicitly and generally disentangle risk aversion from intertemporal elasticity of substitution in the decision maker's preferences. Yet, with Lau's concept one arrives at basically the same results (Result 1 in Section 4) as with our simpler concept.

fraction of debushing was exogenously set to zero in (11), (6) and (8) ($\kappa = 0$, “not debushing”).

Let $\mathbf{v} \in [0, 1]$ denote the constant relative fraction of net income that a debushing farmer is willing to pay in each year for maintaining the rangeland’s carrying capacity on the debushed level. We can determine \mathbf{v} by setting equal the expected utility of the optimally debushing farmer, taking into account the constant relative fraction of income to be paid in each year, and the maximal expected utility of the non-debushing farmer:

$$\mathbb{E} \left[\sum_{t=1}^T \frac{[(1 - \mathbf{v})y_t]^{1-\theta} - 1}{(1 - \theta) \cdot (1 + \delta)^{t-1}} \right] = \mathbb{E} \left[\sum_{t=1}^T \frac{(y_t^0)^{1-\theta} - 1}{(1 - \theta) \cdot (1 + \delta)^{t-1}} \right]. \quad (20)$$

Solving for \mathbf{v} yields:¹⁰

$$\mathbf{v} = 1 - \left(\mathbb{E} \left[\frac{\sum_{t=1}^T (y_t^0)^{1-\theta} / (1 + \delta)^{t-1}}{\sum_{t=1}^T y_t^{1-\theta} / (1 + \delta)^{t-1}} \right] \right)^{\frac{1}{1-\theta}}. \quad (21)$$

This is a risk-averse farmer’s willingness to pay for debushing, i.e. for maintaining the rangeland’s carrying capacity on the debushed level at constant annual marginal costs of C , expressed as a constant relative fraction of net income to be paid in each year. The expected net present value of debushing, V , is then given by the expected net present value of the payment stream that results if in each year a constant fraction \mathbf{v} of annual income y_t is paid:

$$V = \mathbb{E} \left[\sum_{t=1}^T \frac{\mathbf{v}y_t}{(1 + \delta)^{t-1}} \right] = \mathbf{v} \mathbb{E} \left[\sum_{t=1}^T \frac{y_t}{(1 + \delta)^{t-1}} \right] = \mathbf{v} Y_E, \quad (22)$$

where \mathbf{v} is given by Equation (21). The higher V , the more a farmer prefers debushing over not-debushing.

¹⁰For $\theta = 1$, \mathbf{v} is given by:

$$\mathbf{v} = 1 - \exp \left(\mathbb{E} \left[\sum_{t=1}^T \frac{\log(y_t^0)}{(1 + \delta)^{t-1}} - \sum_{t=1}^T \frac{\log(y_t)}{(1 + \delta)^{t-1}} \right] \frac{1}{\sum_{t=1}^T (1 + \delta)^{1-t}} \right).$$

Simulation and optimization method

To simulate the present model, specific MATLAB (version R2009a) code has been developed. The MATLAB routine *patternsearch*, which is a Direct Search algorithm, has proven to be most efficient and robust for solving the current stochastic, discretized optimization problem. Pattern search operates by searching a set of points called a pattern, which expands or shrinks depending on whether any point within the pattern has a lower objective function value than the current point. The search stops after a minimum pattern size is reached. The algorithm does not use derivatives to determine descent, and so it works well on nondifferentiable, stochastic, and discontinuous objective functions (Audet and Dennis 2003).

To approximate the continuous rainfall probability distribution, $N = 1'000'000$ rainfall profiles over T years have been simulated. The sensitivity analysis is performed on the risk aversion coefficient θ , on the discount rate δ , on the standard deviation of rainfall $Sd[r]$ and the fraction of debushing κ , if debushing is regarded as exogenous variable and not optimally determined by the farmer.

4 Results

Result 1: Debushing increases reserve-biomass variation, income variation and the risk premium of income

Figure 3 shows that the coefficient of variation of both reserve biomass CV_R (Equation 13) and income CV_y (Equation 14) at the end of the time horizon T increases with the fraction κ of debushing for all time horizons ($T = 1$, $T = 5$ and $T = 15$). That is, debushing increases the relative variation of both reserve biomass and income in the future.

The reason is that mean reserve biomass and mean income increase less with the extent of debushing than the corresponding standard deviation. The removal of a fraction κ of bushes raises the actual carrying capacity, which is given by $K - (1 - \kappa) \cdot B$, and consequently the reserve biomass (6) and income (8), which has the following consequences:

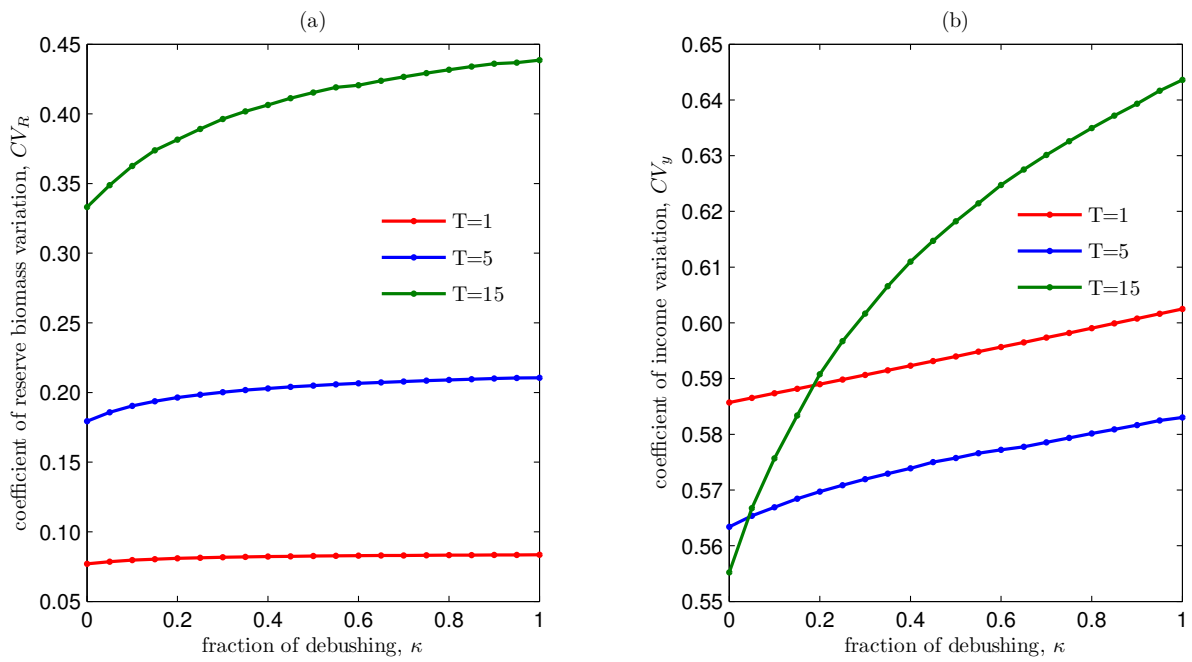


Figure 3: The coefficient of variation of (a) reserve biomass CV_R and (b) income CV_y at the end of the time horizon T as a function of the debushing fraction κ for time-horizons $T = 1$, $T = 5$ and $T = 15$ years. All other parameter values as in Table 1.

it reduces the adverse deterministic impact of the mortality of reserve biomass (i), the impact of grazing pressure on reserve biomass (ii) and partially the stochastic impact of precipitation on reserve biomass (iii). Given that the green biomass G_t (Equation 5) is given as the product of reserve biomass R_t (Equation 6), the rainfall level r_t and a deterministic growth parameter w_G , the increased reserve biomass leads to a higher and more volatile green biomass (iv). Through a feedback mechanism (Equation 6), the higher green biomass G_t generates a more volatile reserve biomass R_t , as the increased green biomass effect is dominating the partial volatility reducing effect of the increased carrying capacity in the reserve biomass process (v). As income y_t (Equation 8) depends linearly on green biomass G_t , all these channels generate finally a *higher* and more *volatile* annual income y_t .

Overall, the coefficient of income variation is a lot higher than that of reserve-biomass variation for all time horizons (Figure 3 a and b). That is, the relative variation of income is a lot higher than that of reserve biomass. This is plausible, because reserve biomass is a stock quantity which buffers fluctuations, whereas income is a flow quantity that is in each year directly determined by, inter alia, stochastic precipitation.

Figure 3(a) shows further that the coefficient of reserve biomass variation is the higher, the higher the time horizon. The reason for this is that the standard deviation of the time-dependent stochastic reserve biomass increases, while the expected reserve biomass decreases, as a function of time. In contrast, Figure 3(b) shows that the coefficient of income variation depends on the time horizon in a non-monotonic manner. Figure 4 shows in more detail how the coefficients of variation of reserve biomass and of income depend on the time horizon if there is no debushing ($\kappa = 0$). While the coefficient of variation of reserve biomass at the end of the time horizon monotonically increases with the time horizon, the coefficient of variation of income at the end of the time horizon depends on the time horizon in a non-monotonic manner: it monotonically decreases for time horizons up to $T = 9$ years, where it reaches a minimum, and from then on monotonically increases with the time horizon.

The reason for this result is the following. As Figure 5(a) shows, for reserve biomass R_T at the end of the time horizon the expected value monotonically decreases and the

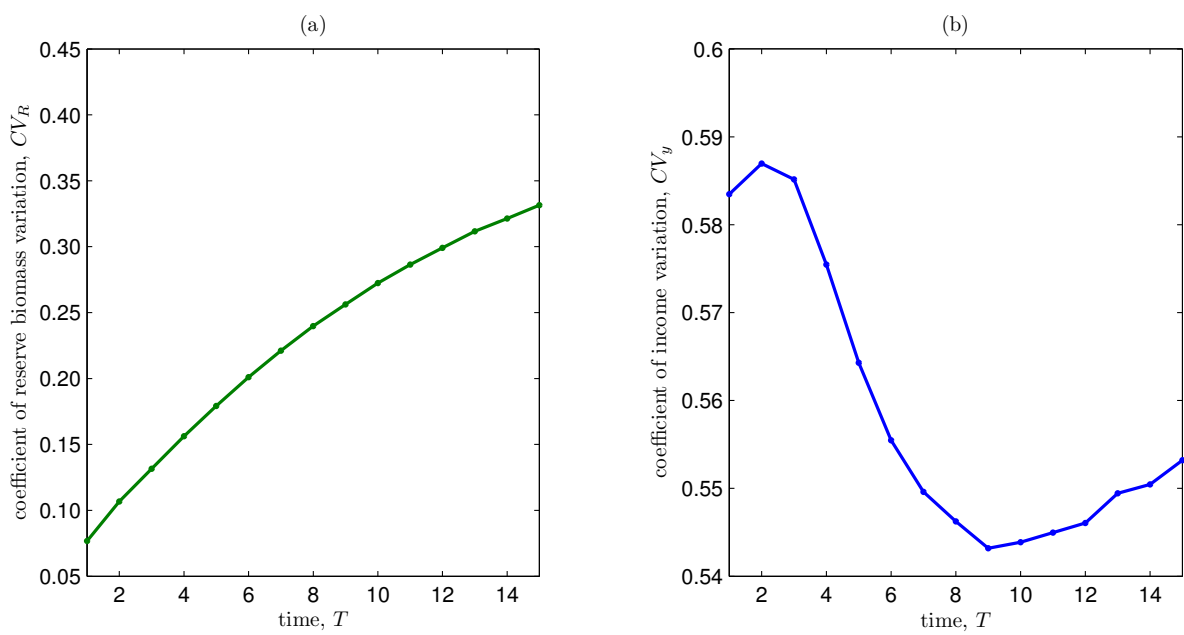


Figure 4: The coefficient of variation of (a) reserve biomass CV_R and (b) income CV_y at the end of the time horizon T as a function of the time horizon T if there is no debushing ($\kappa = 0$). All other parameter values as in Table 1.

standard deviation monotonically increases, so that the coefficient of variation obviously decreases with the time horizon T . In contrast, for the annual income y_T at the end

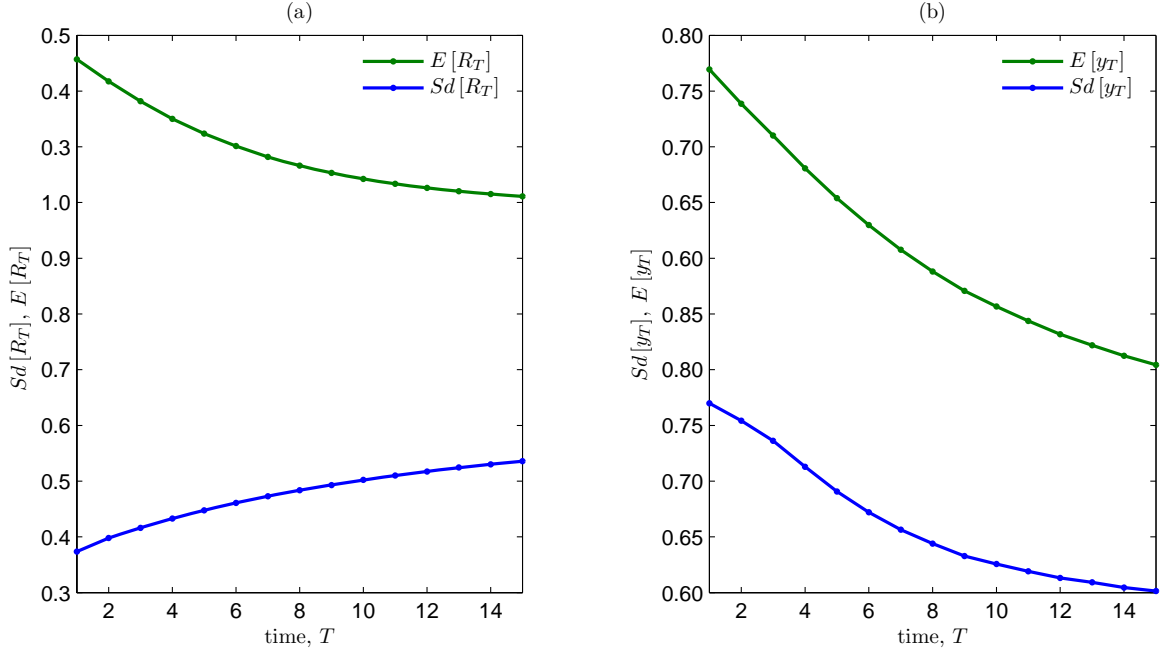


Figure 5: The standard deviation and expected value of (a) reserve biomass R_T and of (b) income y_T at the end of the time horizon T as a function of the time horizon T if there is no debushing ($\kappa = 0$). All other parameter values as in Table 1.

of the time horizon, both the expected value and the standard deviation monotonically decrease with the time horizon T (Figure 5b). However, the *marginal* decrease of the standard deviation is *non-monotonic*, which obviously then leads to a non-monotonic coefficient of variation for income as a function of the time horizon.

Figure 6 shows that the higher the fraction of debushing κ , the higher the risk premium π (Equation 19) of the resulting uncertain income stream. That means, not only does the coefficient of variation of future income increase with debushing (Figure 3b), but also the subjectively valued income risk, which takes into account risk aversion and time preference, increases with debushing. Therefore, debushing does not provide natural insurance to a risk-averse farmer but, just to the contrary, it increases a risk-averse

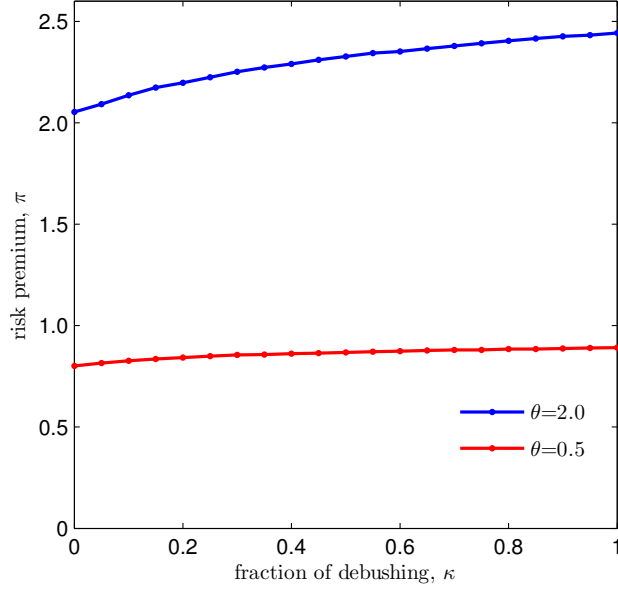


Figure 6: Risk premium π as a function of the debushing fraction κ for two different degrees of risk aversion, $\theta = 0.5$ and $\theta = 2.0$. All other parameter values as in Table 1.

farmer's subjective risk. The reason is that the additional expected income generated by debushing is relatively too low to compensate for the increased income volatility, which leads to an increasing risk premium.

Figure 6 also shows that the risk premium as well as the increase in the risk premium are the higher, the higher the degree of risk aversion θ .

Result 2: Debushing increases the expected net present value of income and the willingness to pay for debushing

Figure 7 shows (a) the expected net present value of income Y_E (Equation 12) and (b) the willingness to pay for debushing V (Equation 22) as a function of the fraction of debushing κ for two different degrees of risk aversion, $\theta = 0.5$ and $\theta = 2.0$. The maximum of the expected net present value of income is reached at $\kappa^{\max} = 0.95$ for $\theta = 0.5$ and $\kappa^{\max} = 0.9$ for $\theta = 2.0$ (Figure 7a). The reason that expected net present value of income reaches a maximum at some level of debushing is a combination of the

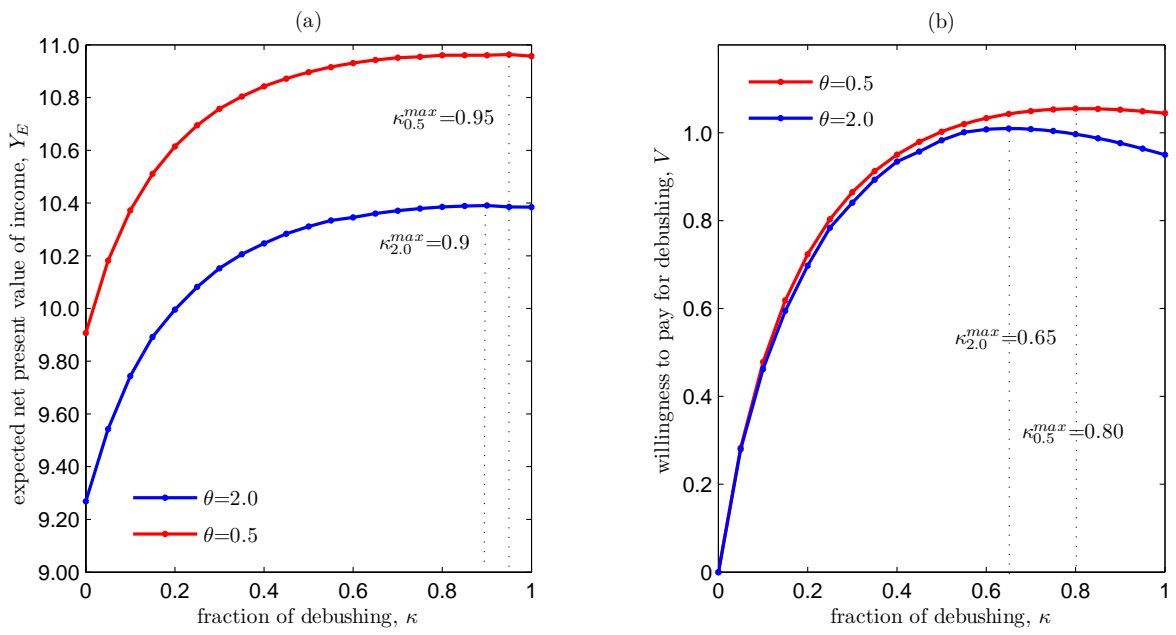


Figure 7: (a) Expected net present value of income Y_E and (b) willingness to pay for debushing V as a function of the fraction of debushing κ for two different degrees of risk aversion, $\theta = 0.5$ and $\theta = 2.0$. All other parameter values as in Table 1.

following two features: (i) debushing comes at constant marginal costs, so that the total costs increase linearly with the level of debushing; (ii) gross income increases with the level of debushing, but less than linearly because of how ecosystem dynamics translates stochastic precipitation in green biomass.

The maximum of the willingness to pay for debushing is reached at $\kappa^{\max} = 0.8$ for $\theta = 0.5$ and $\kappa^{\max} = 0.65$ for $\theta = 2.0$ (Figure 7b).¹¹ For all levels of debushing κ , the higher the degree of risk aversion θ , the lower the expected net present value of income and the lower the willingness to pay for debushing.

Result 3: Debushing implies more conservative grazing management

Figure 8 shows the optimal grazing management strategy, i.e. the optimal fraction of resting γ^* (Figure 8a) and the optimal rain threshold \underline{r}^* (Figure 8b), as a function of the fraction of debushing κ for two different degrees of risk aversion, $\theta = 0.5$ and $\theta = 2.0$. The higher the level of debushing, the more conservative the optimal grazing management strategy, i.e. the higher the optimal fraction of resting γ^* and the lower the optimal rain threshold \underline{r}^* .

There are two reasons behind this result. First, debushing generates a higher expected income, and with a higher expected income, a farmer rests more. Second, debushing increases income risk in the sense that it increases the risk premium (Result 1), and conservative grazing management provides natural insurance in the sense that it reduces the risk premium (Quaas et al. 2007, Quaas and Baumgärtner 2010), so that a risk-averse farmer will counteract the increased risk from additional debushing by applying natural insurance through more conservative grazing management. For a higher degree of risk aversion ($\theta = 2$), the grazing management strategy is therefore more conservative than for lower risk aversion ($\theta = 0.5$).

¹¹Note that the fraction of debushing κ^{\max} which maximizes the expected net present value of income Y_E or the willingness to pay for debushing V will, in general, not coincide with the optimal debushing fraction κ^* that maximizes the expected intertemporal utility U (Equation 9).

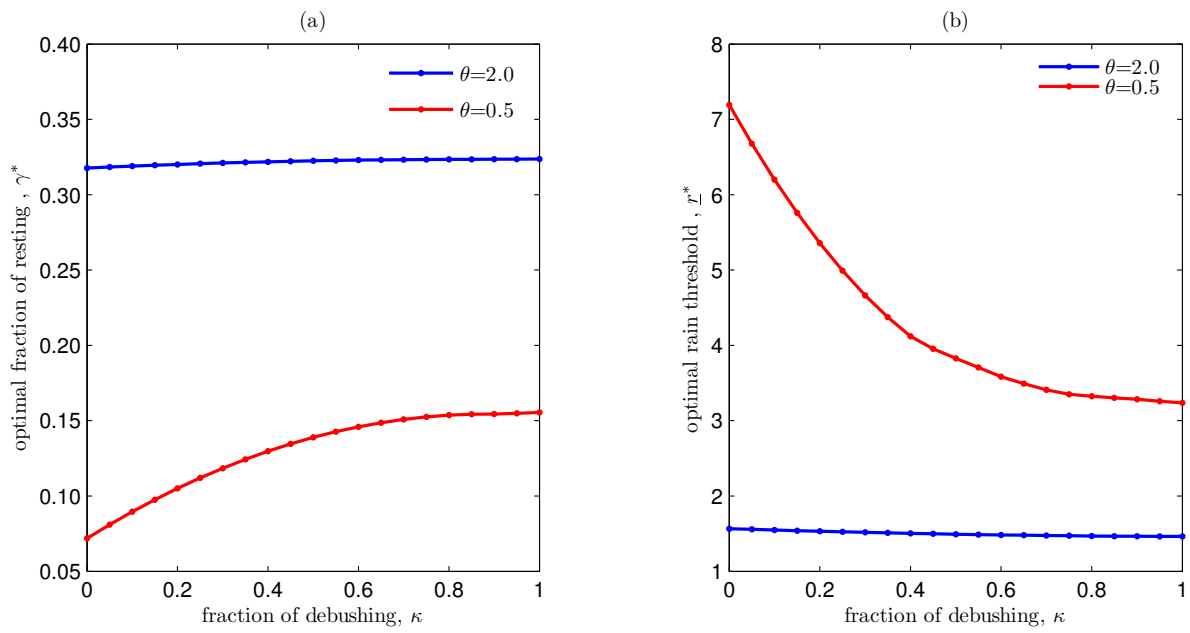


Figure 8: The optimal grazing management strategy, i.e. the optimal fraction of resting γ^* (a) and the optimal rain threshold \underline{r}^* (b), as a function of the fraction of debushing κ for two different degrees of risk aversion, $\theta = 0.5$ and $\theta = 2.0$. All other parameter values as in Table 1.

Result 4: Risk aversion, rainfall volatility and discounting decrease the optimal fraction of debushing

Figures 9 and 10 show the optimal fraction of debushing κ^* as a function of the degree of risk aversion θ (Figure 9a), of the standard deviation of rainfall $Sd[r]$ (Figure 9b), and of the discount rate δ (Figure 10). Apparently, risk aversion, rainfall volatility and

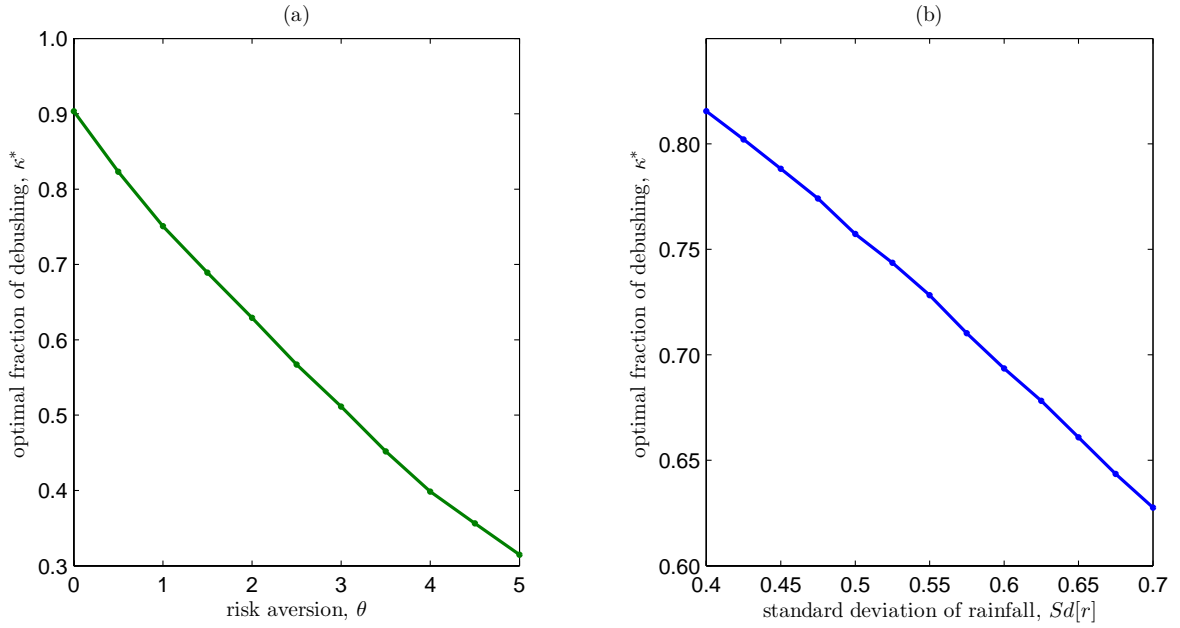


Figure 9: The optimal fraction of debushing κ^* as a function of the degree of risk aversion θ (a) and of the standard deviation of rainfall $Sd[r]$ (b). All other parameter values as in Table 1.

discounting all decrease the optimal fraction of debushing.

Figure 9(a) shows that a higher degree of risk aversion θ implies a lower optimal fraction of debushing κ^* . The reason is that a higher fraction of debushing implies a higher income volatility, which a more risk-averse farmer seeks to avoid more strongly. Figure 9(b) shows that under increased environmental risk (as measured by a higher standard deviation of precipitation $Sd[r]$), the optimal fraction of debushing κ^* decreases. The reason is that a higher precipitation volatility generates a more volatile reserve biomass, a more volatile income, and, eventually, a higher risk premium. As

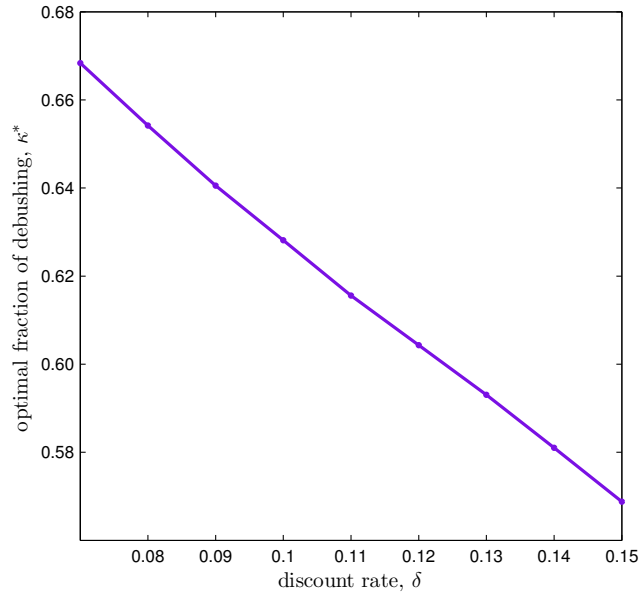


Figure 10: The optimal fraction of debushing κ^* as a function of the discount rate δ . All other parameter values as in Table 1.

already seen, the risk premium can be reduced by reducing the fraction of debushing (Result 1). Therefore, a risk-averse and optimizing farmer reduces the fraction of debushing to counteract the increase in income risk resulting from higher environmental risk. Figure 10 shows that an increase of the discount rate δ leads to a lower optimal fraction of debushing κ^* . The reason is that a higher discount rate decreases the investment motive for debushing, as future income streams are weighted less in comparison to present ones.

5 Discussion and conclusion

We have analyzed the role of debushing for a farmer's income in a stochastic ecological-economic model of grazing management in semi-arid rangelands. Starting from a standard rangeland-management model, we have modeled debushing as a one-time action that permanently increases the carrying capacity of the pasture and carries annual costs. As for the farmer's preferences, we have assumed constant relative risk aversion, a con-

stant rate of pure time preference, and maximization of expected intertemporal utility.

Our central result is that debushing increases the volatility of both reserve biomass and income in the future, and it increases the risk premium to a risk-averse farmer, that is the valued subjective intertemporal income risk. Yet, it also increases the expected net present value of income. The optimal extent of debushing for a risk-averse farmer is thus determined from balancing these two impacts. We find that the higher the farmer's risk aversion or impatience, and the higher the environmental risk (measured as the standard deviation of precipitation), the smaller the optimal fraction of debushing. We further find that the higher the fraction of debushing, the more conservative is the optimal grazing management strategy, that is the more resting is optimal.

The main driver of these results is that in our model debushing causes, first of all, an increase in the carrying capacity of the pasture and, consequently, an increase in the mean level of reserve biomass. But then, the linear dependence of the green biomass on the reserve biomass leads to a higher green biomass volatility if the reserve biomass gets larger. For high levels of debushing, the increase in the volatility of green biomass production (and, hence, income) dominates over the increase in the mean level of green biomass production (and, hence, income), so that a risk-averse farmer will find some limited extent of debushing to be optimal.

Another determinant of the optimal extent of debushing is the structure of costs. While we have assumed constant marginal costs of debushing, there exists in general no interior solution to the optimization problem if – *ceteris paribus* – the marginal costs are zero. In that case, it would for a wide range of parameter values be optimal to fully debush the pasture.

Our results have high relevance for environmental and development policy in regions where farmers face high environmental risk, e.g. from uncertain precipitation, and rely on their farming practices for natural insurance, e.g. through conservative grazing management. Here, we have shown that while debushing will improve the (expected) state of the environment, it also imposes an income risk on the farmer. A risk-averse and optimizing farmer will therefore debush only to a limited extent. It would therefore be desirable, from a combined environment-and-development perspective, to supply farmers

with adequate means of actuarially fair financial insurance against their environmentally induced farm-income risk. For, farmers with access to such financial insurance would have an incentive to debush and thus improve the state of the environment, and would – for the same level of expected monetary income – enjoy a higher level of expected welfare.

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